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"Data models for terrain representation"

Introduction

Terrain data can be stored in a variety of data structures, data models and data formats. A data structure is the internal representation or machine storage method, while a data format is the actual specification of the data structure, and provides a scheme for the systematic storage of data for transfer, processing and retrieval. A data model instead is a more complex entity that, in this tentative definition, represents the phenomenon of terrain according to its ontology (e.g. inherent characteristics) and it is therefore at a more abstract and conceptually higher level than the other two. More specifically a data model of terrain implies a continuous view of the surface under consideration (Kumler 1992). The view employed here considers all terrain representations first as data structures (because they are physically stored as structures in bits and bytes) and then, in their continuity implications, as data models. In other words, different data structures imply different surface characteristics, beyond the data that they actually contain (e.g. contour representation implies a smooth terrain). The objective of this paper is to review the major and emergent data structures for terrain representation, their data models implications and a sample of the formats in which they are distributed.

According to Mark (1979) the source of the data structure should be the phenomenon that needs to be represented, and not an expression of the criteria for efficient storage. For the geomorphologist, the ideal representation is a contiguous, non-overlapping set of polygons with boundaries having geomorphic significance: a landform-units approach, employed in order to capture terrain as a result of a land forming process. Surveyors would prefer terrain as a polyhedron with adjustable facets size so that the precision of the instruments would fit and accommodate the measurements. Photogrammeters and cartographers are more removed from the causes of the phenomenon and prefer contours and abstract representation. Mathematicians would instead employ a particular mathematical surface. This survey will investigate how these different data models are captured by the different data structures reviewed here.

This paper will first present the two data structures of DEMs (Digital Elevation Models) and TINs (Triangulated Irregular Networks), considered in their specific data model implications for terrain representation. Digitized contours will be considered not only as a source data structure for obtaining other representations like the DEM, but also a data model in its own right which defines a terrain with particular characteristics. The contribution of 3D GIS to the representation of terrain surfaces is considered in relation to voxels, a data structure optimized for visualizations, and spline curves, which similarly to contours they serve the dual role of data structures for interpolation and data models that imply a particular class of terrain shapes. VRML (Virtual Reality Modeling Language) is finally introduced with a special attention to its capability of representing surfaces using data structures that are virtual counterparts of DEMs and TINs. In the appendix two data formats, the USGS DEM and the PDS MDIM for planetary topographic data, will be reviewed.

Digital Elevation Models (DEM)

DEM data are records of terrain information in the form of a sampled array of data at regularly spaced horizontal intervals (USGS 2000). Grids are structured in matrices of values that record topological relations between data points implicitly. The matrices are accompanied by information concerning geographical registration and accuracy assessment (see Appendix I – USGS DEM).

It is a simple data model since it can be stored in arrays, basic internal computer data structures, and the algorithms used to process them tend to be straightforward. One drawback of the uniform coverage of sample points is that point density of regular grids is not adaptive to the complexity of the relief, and more data points are needed to represent the terrain to a required level of accuracy (Weisbel and Heller 1991)

The altitude matrix data structure do not conform with any of the different professionals' views presented in the introduction, because it is not based on a landform subdivision of the terrain, nor it is abstract or structured on mathematical functions. The view of Mark (1979) is that it does not capture appropriately the phenomenon of terrain and is used only for convenient machine storage.

In the production of DEMs, cartographic sources, like DLGs (see below "Digitized contours") hypsography and scanned contours, are used more widely than photogrammetric sources, which are based on the use of aerial photographs with manual or automated stereoplotting (USGS 2000). In particular, three main classes of source elevation data have been recognized, namely surface-specific point elevation data (obtained through ground survey and GPS), contour and stream line data (digitized sources), and remotely sensed data (aerial photography, laser and synthetic aperture radar).

DEMs are generated through interpolation and filtering of such variety of data sources, and recent developments in locally adaptive gridding techniques have enhanced the matching of grid structures to terrain characteristics, beyond the limitations of uniform sampling. Regular-grid DEMs are used primarily in environmental modeling and natural resource assessment, and can be integrated with remotely sensed environmental data sources (Hutchinson & Gallant 1999).

Triangulated Irregular Network (TIN)

The Triangulated Irregular Network (TIN) is constituted by points that are distributed irregularly on the surface, and are linked together by straight lines so that a contiguous, non-overlapping set of triangular elements is formed (Mark 1979). Structural features (such as ridges and peaks) can be accommodated by the data structure explicitly, while topological relations must be computed and stored explicitly, making them more complex and difficult to handle (Weisbel and Heller 1991).

TIN's data structure has been developed for two main reasons. First of all, since topographic surfaces are non-stationary, the roughness of the terrain changes from one terrain to another. While a regular grid is highly redundant in smooth terrain, it was necessary to have a data structure able to capture the surface with an adaptive number of points. Second, the terrain model must resemble closely the real world. For example, simple spot heights are not representative of a real world structure beyond their specific ability of defining altitudes at points. A terrain model, in other words, should contain those features that are the natural units of the analysis. These features are topographic elements like ridges, channels, peaks, passes, which have high information content in both contexts of analysis (data model) as well as storage optimization (data structure) (Peucker et al 1978).

With these two purposes in mind the TIN was devised, based on coordinate random but surface specific sample points whereby the location of points is determined by the surface, and their attributes include the explicit index of neighboring points for providing a substitute to the addressing matrix algebra of grids. Points constitute the primary data structure, while the secondary data structure represent features of the terrain as holistic objects. The specific data structure of TINs includes 6 pointers per triangle (3 nodes and 3 triangles), with each point having coordinates, labels, attributes, number of neighbors, and a pointer to the neighbors list (Peucker et al 1978).

The generation of a TIN includes the selection of points (error prone manual selection relying on contours or automated selection) and the connection into facets. The definition of the secondary data structure is based on the extraction of a skeleton of surface specific points and lines. Points and linear features are then connected in a Delunay triangulation. Points can be added to the representation if the discrepancy between the TIN and the original is above a given threshold (Peucker et al 1978). Possible variations on the TIN model include triangulation criteria and the form of the surfaces between triangle, planar or higher order (Mark 1979).

TINs have been used for contouring and interpolation since the early 1960s and they have been applied to terrain modeling problems (Weisbel and Heller 1991). The immediate applications of the data structure include sequential processing, search, surface intersection, and specifically hill shading, slope mapping, contouring, profiling and line of sight. (Peucker et al 1978). TINs are mostly used as data reduction tools, especially useful in visualization applications (Weisbel & Heller 1991)

As a form of terrain representation, TINs reflect better the variable density of points necessary to capture the roughness of the terrain. Also the TIN is based on a concept that is relevant to the phenomenon view of geomorphologists and surveyors (that is, landform-based subdivision and adaptability to measurement methods) (Mark 1979), and from that point of view is more appropriate than grids for terrain representation. On the other hand the advantage of TINs over grids is not obvious. TINs do not represent properly terrain shape parameters such as slope and curvature (Hutchinson and Gallant 1999), a possibility offered easily by grid data. From a data storage point of view, TINs are efficient in representing terrain only if terrain itself is suitable for a reduction to a structure of triangular facets. Greatly varying terrain, with no areas of uniform surface, is better represented by a grid, while TINs are suitable for terrains characterized by fluvial processes and mass movement. Also the data structure is more complex than a grid data structure that adapts easily to array representations. In conclusion, there is no implicit general superiority of TINs over grids, but only application specific issues of convenience to be considered (Kumler 1992).

Digitized contours

The use of contours for constructing DTMs is widespread but present some problems related to production and representation. In fact contours are mainly a form of terrain visualization and they are not particularly useful as a scheme for numerical surface representation. In particular, they present oversampling along contours, undersampling across contours (especially in areas of low relief), and a variety of errors introduced in production and mapmaking (Weibel & Heller 1991).

Elevation contours are a compromise method of obtaining DTMs for medium or small scales, and they are still the main data source for the interpolation of DEMs. They can reflect surface structure even if they present inherent sampling biases, requiring a high quality interpolation technique. Many contour data have been digitized from existing topographic maps, which in some parts of the world are the only source. The validity of contours as a form of representation is supported by the fact that they are able to implicitly encode a number of terrain features like ridges and points on stream-lines, which would not be captured by other data structures (e.g. grids) (Hutchinson & Gallant 1999).

The validity of contours is also dependent on the quality of the interpolation algorithm used to extend the information they contain. Many researchers prefer contour specific algorithms instead of general interpolators. An option is to grid DTMs with cubic interpolation along straight lines for extending the contour line information to the area in between. Alternatively, drainage enforcement algorithms can be used, which first remove sinks and pits and then calculate ridge and stream lines out of points of maximum curvature (for a more reliable interpretation of the fine structure of contours).

Contours differ from other forms of representation because they imply a degree of smoothness of the underlying terrain. Concerning the previous discussion about data redundancy of grids versus TINs, it can be said that contour encoding can be adaptive to terrain variability but it is very inefficient for many types of computations (Peucker et al 1978).

The USGS format for digitized contours is the DLG (Digital Line Graph). A DLG is a representation in vector form of the line information found on USGS topographic maps. Each unit of representation contains a specific category of data, and the vectors are tagged with attribute codes defining the category. Topological structure is provided to preserve the spatial relationships inherent in the source map. Considering large-scale DLG units for elevation data, the hypsography category includes contours and supplementary spot elevations (USGS 2000).

The contribution of 3D GIS

This section will consider two elements of 3D GIS that are relevant to the representation of terrain surfaces, namely volumetric elements (voxels) and mathematical spline curves.

While voxels are inherited from 3D mainly for purposes of visualization, splines can be considered amongst interpolation methods. Both are forms of representation on their own right and will be considered here as such.

Voxels are the basic unit of expression of a raster approach to data representation. In a 3D raster they constitute the analog of the pixel element of 2D graphics. More specifically, we can obtain a voxel by subdividing regions of 3D space according to an octree structure into octants (the corresponding elements of the quadrants of quadtrees). Each octant stores eight data regions, and the volume subdivision proceeds hierarchically until individual unit elements (the voxels) are reached (Hearn &Baker 1986).

Several examples of landscape animation have made use of surfaces represented as 3D voxels (Cohen & Gotsman 1994) (Cohen-Or & Rich 1996) (Wan et al. 1999). In all cases voxels constituted an intermediate data structure optimized for visualization, which mediated the original grid-based terrain representation with the final output of real-time perspective graphics. The approach used was called of “volume graphics”, by which a 3D raster of voxels was used to represent the surface after 3D rasterization (voxelization), and each voxel value was considered as an independent variable having an elevation value and optical parameters. The flythrough in (Cohen-Or & Rich 1996) is an example of the application of this form of representation whose adoption is motivated by performance requirements. It is in fact optimized for visualization because allows higher resolution than other approaches, an easier texture mapping (textures are preprocessed and there is no overhead during real-time rendering), and it is complexity independent (incorporates easily other visual elements) (Wan et al. 1999)

In the broader context of terrain representation, voxels constitute a particular data structure that resembles the raster grid approach of 2.5D graphics, and shares with it some aspects and limitations (e.g. implicit topology and redundancy). It is mainly a technical solution to the problem of terrain visualization, but probably it is not particularly influential as a model of terrain, because it falls into the broader category of raster approaches to representation.

Terrain can be represented by mathematical curves that model the behavior of surfaces according to controllable parameters. According to Mark (1979) this would be the preferred form of terrain representation by mathematicians. Here, spline curves are not considered simply as tools for interpolation, but as forms of representation that imply particular terrain characteristics.

NURBS (Non-Uniform Rational B-Splines) are parametric polynomial functions that are used to structure points into a single 3D model by assembling multiple components of curves, with the assumption of continuity of curvature (Raper and Kelk 1991). Defining a surface using NURBS would enforce constraints on the actual shape of the terrain. This would result inappropriate for capturing the characteristics of topography, even if it is an acceptable approximation for computer graphics applications.

Smith & Wessel (1990) used splines in tension as a gridding method based on the minimization of curvature. They avoided undesired oscillations by relaxing the criteria of exact interpolation or minimization of total curvature. Modifying the tension parameter affects the oscillations and the inflection points, generating considerably different landscapes.

Mitasova et al. (1996) used regularized splines with tension to construct a high resolution model of terrain from contour data, judged better than the original 30m DEM that had systematic errors and artifacts, for modeling erosion and deposition. The splines were constructed based on the minimization of a general smoothness functional, and included the use of tension and smoothing parameters. In particular tension was used to tune the interpolant from the two extreme forms of thin steel plate to membrane, to minimize overshoots and artificial pits, while smoothness controlled the closeness to original data. Mitasova & Mitas (1993) used the tension parameter for changing the character of interpolation, and it gave the flexibility for obtaining high levels of accuracy. Segmented processing allowed for a close modeling of the surface by incorporating local behavior.

In conclusion the behavior of splines is flexible thanks to tension and smoothness parameters, but it does not necessarily follow topographic criteria, which need to be explicitly enforced. As a terrain representation tool, splines can be adapted to any terrain with some approximation, but even local methods would find difficulties in representing very rough terrain where the uniform and smooth properties of the curves are not suitable representations.

Virtual Reality Modeling Language (VRML)

It is possible to represent terrain using tools dedicated to the development of virtual worlds. In the new context of virtual reality, terrain becomes an interactive element in a dynamic visualization. Virtual Reality Modeling Language (VRML) is the language that allows the definition of a virtual environment. A Web browser equipped with the proper plug-in interprets the VRML code and then visualizes the representation through an interface, so that the objects are generated in real-time from their specifications. A virtual environment is coded as a set of nodes that define its structural properties (for example the constituent objects), while each node is further specified by fields and field values, the actual parameters of the representation (Nadeau 1999).

The geometry of a surface is represented in VRML by a node called ElevationGrid, a relatively recent addition to the language that in practice consists in a more compact way of representing terrain than the more general IndexedFaceSet polygonal node. The latter is a specification of 3D objects obtained by defining 3D coordinates and indexing them in individual polygons that represent the facets of the object. A terrain representation made using an IndexedFaceSet specifies the surface through explicitly defining the individual facets. IndexedFaceSet may resemble a TIN structure, especially for the subdivision in triangular facets, but the topology information is limited to the composition of facets with vertices.

ElevationGrid has been created as an appreciation of the common use of terrain in virtual worlds, as well as in recognition of the wide applicability of 2.5D grids to many kinds of phenomena. ElevationGrid is essentially a 2D grid (a height field) defined by the number of elements for the two axes, by the horizontal spacing between samples, and by a scalar array of floating point heights. The sample points represent in 3D the actual vertexes of quadrilaterals that are then broken down in triangles by the visualization procedures operated by the specific browser (Carey & Bell 1997). However, ElevationGrid has several limitations, including the assumption of a flat plane from which elevations are calculated, a limitation that is not convenient for geographic purposes when curved planets are involved, and is therefore useful only for local areas. In fact in actual large-scale geographical applications (Reddy et al. 1999) (Rossi & Spagnuolo 1999) the more general IndexedFaceSet is used instead.

As mentioned earlier, IndexedFaceSet can be considered a form of TIN, while ElevationGrid is a traditional elevation raster. Issues of data redundancy for grids expressed earlier apply to ElevationGrid, but this does not extend to the implementation details of the data structure which is in fact more efficient than IndexedFaceSet in terms of storage and processing.

Reddy et al. (1999) present an approach for terrain representation that uses an optimized version of VRML. It allows a multi-resolution and real-time rendering of large terrain datasets. The world coordinates are converted from the projection of the source dataset into the geocentric reference system of VRML. The system maintains real-time characteristics even with large datasets by subdividing the database in a pyramid of layers at different resolutions. According to the position of the observer it displays the data at the resolution that is relevant and convenient in a trade-off between detail and speed.

The visualization of geographical data on the Web has stimulated the development of additional VRML features that could accommodate for the increased complexity of the data being managed. GeoVRML (www.geovrml.org) is a Working Group of the Web3D Consortium (www.web3d.org) and aims at the development of tools and recommended practice for the representation of geographical data using VRML. The objective is to incorporate geo-referenced data such as maps and 3-D terrain models into the visualization on the Web by using a standard VRML plug-in for a Web browser. The representation of terrain allowed by GeoVRML is an extension of the common VRML functionality, and in fact it is based on a run-time environment based on additional Java classes. There are several features that are worth noticing, including the possibilities for georeferencing and the in-built management of Level of Detail (LOD) for multi-resolution databases. In relation to terrain representation it is particularly interesting the GeoElevationGrid node, which extends the standard ElevationGrid by introducing the specification of coordinate systems, such as GDC lat/long, UTM easting/northing and GCC (based on planetary offset from WGS84 ellipsoid).

Conclusions

From the survey of the data structures and models it emerges that there are two main representations, regular-grid DEMs and TINs, on which alternative structures such as voxels and VRML are based. Contours and splines serve the dual role of data models (characterizing terrain with a specific signature) and intermediate data structures for interpolation towards a DEM. The opinion of Mark (1979) according to which grids are the least appropriate data structure is in contrast with the centrality of grids as preferred storage structures of terrain information collected through interpolation from a variety of data sources. It might be that the ease of conceptualization, modeling and use through processing is still a powerful attraction that downplays the issues of redundancy and lack of encoding of explicit entities of terrain form.

The in-depth comparison between TINs and DEMs is a separate issue and has been carried out elsewhere (Kumler 1992), but it does not seem that there are convincing clues supporting the superiority of TINs over grids especially from a point of view of storage efficiency. From a data model point of view, TINs are suitable for geomorphologists and surveyors, because they represent terrain according to a structure that adapts to landform units and incorporates explicitly shape information (Mark 1979).

According to Weibel & Heller (1991), no data structure is superior for all tasks of digital terrain modeling, and one capability that might become necessary is to switch representation. The basic representation should be based on a data structure that is general enough for incorporating a variety of uses. Specific representations would derive from the base of the original representation.

Interesting prospects come from terrain representation in virtual worlds. There are no striking changes from the original non-virtual representations, but it is interesting to note how, from a data structure point of view, the creators of VRML tried to improve the efficiency moving from TIN like representations to grids, introducing both advantages and disadvantages of such data structure. Another data structure used for similar reasons of efficiency is the voxel representation, which inherits the grid characteristics in a 3D context and links terrain representation to terrain visualization in a synergy that is not attainable otherwise.

In conclusion the multiple data structure envisioned by Weibel & Heller (1991) would allow switching from a virtual world optimized representation, to a geomorphologically sound, surface specific structure. This would support an interaction with the terrain that goes beyond the seams of the storage method and allows a total experience of analysis, visualization and exploration.

Appendix I – DEM formats

The USGS DEM

The following section is a more in depth description of the USGS DEM formats, based on the USGS Data Users Guide 5 (USGS 1993). The purpose is to convey a precise idea of the national official standard for mapping and terrain representation, in an attempt to find a connection between data structures (in this case, the gridded DTM) and data formats (the actual organization of data storage).

The USGS DEMs come in 5 different formats that are identical in logical data structure but differ in sampling interval, geographic reference system, areas covered and horizontal and vertical accuracy. An individual DEM file is organized into a series of three ASCII records, named A, B, and C. The A record contains information defining the general characteristics of the DEM, including name, boundaries, elevation range, number of B records (i.e., description of the actual elevation matrix), and projection parameters. Each B record consists of an elevation profile with associated header information, while the C record contains accuracy data relative to the dataset. There are one A and C records per file, while every elevation profile is represented by a separate B record.

The five formats are the 7.5 minute DEM, the 7.5 minute Alaska DEM, the 15 minute Alaska DEM, the 30 minute DEM (also known as 2 arc-second DEM) and the 1 degree DEM.

7.5 minute DEM. Projected in UTM coordinates, it has a 30m data spacing and covers the area of a standard 7.5 minute map series. Uses the reference datum of NAD 27 or NAD 83 and has data ordered from south to north in east to west profiles, which have a different numbers of elevations due to difference between grid north and true north. It is produced from digitized contours or automated or manual scanning of aerial photography. The production normally involved 4 processes: the use of the Gestalt Photo Mapper (now discontinued); manual profiling from photogrammetric stereomodels; stereomodel digitizing of contours; and derivation from DLG (see below) hypsography and hydrography.

7.5 minute Alaska DEM. It has a lat/long resolution of 1x2 arc seconds, uses datum NAD27 or NAD83 and it is produced to match the spatial format of 1:24000 and 1:25000 scale source map contours. The production involves raster to vector digitizing and then gridding of contours.

15 minute Alaska DEM, It has a lat/long resolution of 2x3 arc second and correspond to the 1:63360 map series, using NAD27 or NAD83. It corresponds to four different map quadrangle sizes according to latitude. The profile is made of 451 elevations. It is produced from combination of digitized hypsographic and hydrographic data from original 1:63360 scale maps.

30 minute DEM. The resolution is lat/long 2x2 arc seconds, and it corresponds to half of a 1:100000 map, using NAD27 or NAD83. It is composed by a total of 451x2 (twice the 15 minutes DEM) profiles. It is produced by derivation from DLG contours from maps from a scale of 7.5 minutes to 30x60 minutes, or by resampling from a DEM having a sampling interval equal or less 2 arc seconds.

1 degree. It has a lat/long resolution of 3x3 arc seconds, and covers half of the 1x2 degree map series. It uses the datum of World Geodetic System 1972 (WGS72) or WGS84. The profile is composed by 1201 elevations. The spacing between profiles varies by latitude, from 6 arc seconds (601 profiles) between 50 deg and 70 deg N, to 9 arc seconds (401 profiles) for Alaska north of 70 deg. Selected 1 degree DEM are regridded from 7.5 minute and 30 minute by NIMA (National Imagery Mapping Agency). It is obtained from photographic sources (integrated by additional information about hypsography) and cartographic sources (digitized contours)

The availability of data in these formats is indicative of the extent of the USGS mapping effort. The coverage of the 7.5 minute DEMs is strongly uneven across states. For 1 degree DEMs the coverage is complete for all contiguous United States, Hawaii, portions of Alaska, Puerto Rico and Virgin Islands. 7.5 minute Alaska DEMs have a complete coverage. The 30 minute DEM is a relatively new product, and at the moment of writing 35 % of the coverage is completed, and the intention is to complete the remaining in 2-3 years (USGS 2001)

DEM Accuracy

The issue of data accuracy becomes important when the DEM data are used in terrain modeling. Horizontal accuracy in a DEM depends on the horizontal spacing of the elevation matrix, since original terrain features are generalized and reduced to grid nodes spaced regularly in the horizontal plane. This generalization reduces the ability to recover information about positions that are separated by less than the internal spacing, which results in the filtering of the surface. Vertical accuracy depends on spatial resolution, source data, collection procedure and digitizing systems. The method for determining accuracy is based on the computation of the root mean squared error (RMSE) between the DEM and true elevations on maps, with test points being representative of the terrain.

The record C of a USGS DEM contains accuracy information. According to accuracy level, the DEMs are classified in three groups. Level 1 is reserved for 7.5 minute DEM, has an RMSE of 7 meters (with a maximum error of 15m). The vertical error tolerance is equal to 50 meters. Level 1 includes 7.5 minute DEMs regridded to 30 minute DEMs.
Level 2 is for DEMs obtained from digitized (maps and photographic) hypsographic and hydrographic data, and has an RMSE of half contour. Level 3 is for data derived from DLG data such as hypsography and hydrography, plus additional information like transportation features, with an RMSE of one third contour interval. The majority of 7.5 minute DEM is level 1. The actual priority of production is given to Level 2 data, while there is no level 3 data actually produced by USGS. 30 minute DEMs obtained from contour data are level 2, those from 7.5 are level 1. NIMA 1degree DEMs can be classified as level 3 for the hypsographic information.

PDS MDIM DTM

The PDS (Planetary Data System) MDIM (Mosaicked Digital Image Model) is a data format used for both images and terrain models of the surface of Mars, derived from the data collected by the Viking orbiters and compiled by USGS in map products. The Mars Viking DTM is based on a control net with 5 km horizontal and 1.5 km vertical standard errors, and is projected on a Sinusoidal Equal Area projection. The resolution is computed on planetocentric rather than linear measures, and therefore the range of resolutions goes from 1/256 to ¼ degrees per pixel (in linear units these extremes correspond to respectively 0.231 km and 3.692 km per pixel). The DTM was compiled by manually digitizing the 1:2,000,000-scale series maps produced by USGS at 1 km contour interval, and interpolating using a distance weighting algorithm on contours. The DTMs comes in tiles of 15x15 degrees. The DTM data object is fixed length and is distributed on binary files, containing a descriptive label (resolution, projection, elevation range, etc.), a histogram of the data, and a collection of records containing lines of 16-bit signed integer numbers (Eliason 1992).

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Rossi & Spagnuolo (1999) developed an interactive VRML model for the management of the Coastal zone of the Ligurean coast of Italy through Web and VRML. Use of VRML possibilities for interactively represent datasets.